

AP Calculus AB*Test Review: Basic Integrals*

Find the indefinite integral.

$$1. \int \left(3 + 2x - \frac{4}{x^2} \right) dx \qquad 2. \int (2 \sin x - \cos x) dx \qquad 3. \int 4x \left(\sqrt{x} + \frac{1}{x^2} \right) dx$$

Let $f(x)$ be an **even** function such that $\int_0^4 f(x) dx = 9\pi$. Let $g(x)$ be an **odd** function such that $\int_0^4 g(x) dx = 12$. Evaluate the following.

$$4. \int_{-4}^4 f(x) dx \qquad 6. \int_{-4}^4 [g(x) - f(x)] dx \qquad 8. \int_4^0 [f(x) + g(x)] dx$$

$$5. \int_{-4}^4 g(x) dx \qquad 7. \int_0^4 [3f(x) + 1] dx \qquad 9. \int_4^0 [f(x) - g(x)] dx$$

Evaluate the definite integral

$$10. \int_0^{\pi/4} (3 \cos x - \sin x) dx \qquad 11. \int_0^{\pi/6} (2 - \sin x) dx \qquad 12. \int_1^9 \frac{3\sqrt{x}}{2x} dx$$

Use the table below to evaluate the definite integrals.

x	-2	-1	0	1	2
$f(x)$	-5	8	11	10	12
$f'(x)$	4	-7	-9	1	0
$f''(x)$	-3	-1	0	2	6

$$13. \int_{-2}^0 f'(x) dx \qquad 14. \int_{-2}^1 f''(x) dx \qquad 15. \int_0^2 \left[f''(x) - \frac{2}{3} f'(x) \right] dx$$

16. Snow is accumulating at a rate of $r(t)$ inches per hour. Give units for each of the following.

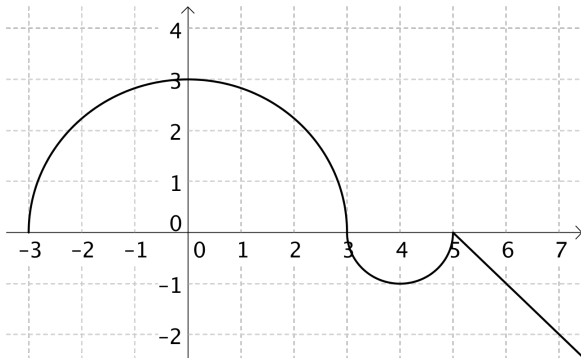
$$(a) r'(t) \qquad (b) \int r(t) dt \qquad (c) \int_a^b r(t) dt \qquad (d) \frac{1}{b-a} \int_a^b r(t) dt$$

17. A particle moves along the x -axis with velocity $v(t) = 1 + 2t - 0.25t^3$.

- Find the average velocity of the particle on the interval $[1, 3]$.
- Find the average acceleration of the particle on the interval $[1, 3]$.
- If the initial position of the particle is $x = -3$, find the position at time $t = 4$.
- Find the total distance traveled from time $t = 0$ to $t = 5$.
- Find all times on the interval $[0, 5]$ when the speed of the particle is increasing.

t (hours)	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
$v(t)$ (km/hr)	0	45	72	88	95	80	62	58	50

18. The table shows the velocity of a vehicle recorded at various times over a two-hour interval.
- Estimate the acceleration of the vehicle at $t = 1$ hour.
 - If possible, find the average acceleration of the vehicle on the time interval $[0, 2]$. If not, explain why the value cannot be determined.
 - If possible, find the average velocity of the vehicle on the time interval $[0, 2]$. If not, explain why the value cannot be determined.
 - If possible, find the total distance traveled by the car during the two-hour time interval. If not, explain why the value cannot be determined.
 - Interpret the meaning of $\int_0^2 v(t) dt$ in the context of the problem. Include units.
 - Interpret the meaning of $\frac{1}{2} \int_0^2 v(t) dt$ in the context of the problem. Include units.
 - Approximate $\int_0^2 v(t) dt$ using a midpoint Riemann sum with four subintervals of equal length.
 - Approximate $\int_0^2 v(t) dt$ using a right-hand Riemann sum with four subintervals of equal length.
19. Find the average value of the function $f(x) = x^2 - \sin x$ on the interval $[0, \pi]$.
20. The graph of $f'(x)$ shown below consists of two semicircles and a line segment.



- Find the exact value of $f(3) - f(0)$
- Find the exact value of $f(5) - f(-3)$
- If $f(6) = \frac{3\pi}{4}$, find the exact value of $f(4)$
- If $\int_0^{10} f'(t) dt = 20$, find $\int_7^{10} f'(t) dt$